

# Immunization Strategy for Multinational Fixed-Income Investments

## **Shmuel Hauser**

Chief Economist, Israel Securities Authority  
Senior Lecturer, Ben-Gurion University

## **Azriel Levy**

Head of Foreign Exchange Research, Bank of Israel  
Adjunct Professor, Hebrew University

## **Uzi Yaari**

Professor of Finance  
School of Business, Rutgers University-Camden

## **Introduction**

Fixed-income asset and liability portfolios held in various currencies by multinational corporations and financial institutions are subject to the combination of foreign currency risk and interest rate risk. Redington (1952) and Bierwag, Kaufman and Toevs (1983) show how to immunize interest rate risk in a fixed-income portfolio invested in a single currency. Their approach is extended here to the immunization problem in the multinational arena, in which interest rate risk and currency risk must be managed simultaneously. The most obvious treatment of this problem is by matching the duration of assets and liabilities separately in each country. Unfortunately, this solution is likely to be prohibitively costly. In this chapter, we explore the conditions under which the multinational firm can dramatically lower hedging costs. Under these conditions, the firm is able to hedge against foreign and domestic interest rate risks through immunization by matching the *overall* duration of the asset and liability portfolios rather than the duration of asset and liability portfolios in any single market. In that setting, the optimal management of the risks of interest rates and foreign currency may be done separately rather than jointly. This chapter extends

the results of Gadkari and Spindel (1989), Hauser and Levy (1991), and Leibowitz, Bader and Kogelman (1993), who show that hedging currency risk converts some or all of the foreign-held claims to synthetic domestic claims.

In the second section of the chapter, we derive the necessary and sufficient conditions under which currency and interest rate risks can be managed separately. In the third section, we present an empirical test of the assumptions underlying those conditions. The final section concludes this chapter.

### **Hedging Interest-Rate and Currency Risks**

For the sake of exposition, the general case of two portfolios consisting of assets and liabilities denominated in multiple currencies is explored below via the simpler but essentially similar case of two-currency portfolios. The analysis takes the view of an American investor for whom both portfolios represent a mix of dollars and yens, assuming for simplicity that the yield curves in the United States and Japan are flat and subject to parallel shifts. The latter assumption has been extensively discussed in the literature and deserves explanation. The definition of Macaulay's duration assumes that the term structure of interest rates is flat; its interpretation as price/interest-rate elasticity assumes further that the magnitude of interest rate changes is infinitesimal. Given the complex stochastic nature of interest rates and the variety of yield-curve shapes and their frequent changes, the use of Macaulay's (1938) duration often fails to achieve a successful hedge through immunization.<sup>1</sup> For example, the presence of nonparallel shifts under immunization designed to control the risk of parallel shifts will cause the value of assets to fall short of the value of liabilities. A great number of contributions in recent years explore risk-reduction methods based on alternative duration definitions and risk-producing stochastic processes—some recognizing the effect of transaction costs. However imperfect, immunization in numerous versions has become a popular technique for hedging interest rate risk in recent years. Its practical value for risk control may be attributed to its simplicity and flexibility in imposing relatively few constraints on the structure of assets and liabilities managed.

The immunization strategy derived in this chapter follows this tradition. We view the choice of a duration measure and underlying interest rate process as a tradeoff among imperfect immunization methods, all of which generate errors, and opt for the traditional simple Macaulay's duration. This choice makes tractable and accessible our derivation of an optimal immunization strategy, but should not be considered an oversimplification of the problem. In view of the alternatives, there is no clear sacrifice in our choice: Studies cited above reveal parallel imperfections in alternative

immunization methods, and specific results of Hegde and Kenneth (1988) show that immunization errors caused by the assumptions of Macaulay's duration are economically insignificant.

## An Immunization Strategy for International Fixed-Income Portfolios

Let

$S$  = dollar/yen spot exchange rate

$F$  = one-period dollar/yen forward exchange rate

$r_d$  = dollar interest rate

$r_y$  = yen interest rate

$A_d, A_y$  = dollar and yen assets, respectively, denominated in domestic currency

$L_d, L_y$  = dollar and yen liabilities, respectively, denominated in domestic currency

$A, L$  = total assets and liabilities

Let the value of the assets and liabilities involved be

$$(1) \quad A_d = \sum_i A_{id}(1+r_d)^{-i}$$

$$(2) \quad L_d = \sum_i L_{id}(1+r_d)^{-i}$$

$$(3) \quad A_y = \sum_i A_{iy}(1+r_y)^{-i}$$

$$(4) \quad L_y = \sum_i L_{iy}(1+r_y)^{-i}$$

where  $A_{id}$ ,  $L_{id}$ ,  $A_{iy}$ , and  $L_{iy}$  are the receipts and payments in period ( $i=1,2,\dots,n$ ), denominated in dollars and yens, respectively, so that

$$A = A_d + SA_y$$

$$L = L_d + SL_y.$$

Without loss of generality, it is conveniently assumed that the investor's net worth,  $N$ , is initially zero, namely  $N=A-L=0$ .

Based on Macaulay's (1938) definition, let  $D_{Ad}$ ,  $D_{Ld}$ ,  $D_{Ay}$ , and  $D_{Ly}$  denote the duration of the dollar and yen assets and liabilities, respectively, where

$$(5) \quad D_{Ad} = [\sum_i A_{id}(1+r_d)^{-i}] / A_d$$

$$(6) \quad D_{Ld} = [\sum_i L_{id}(1+r_d)^{-i}] / L_d$$

$$(7) \quad D_{Ay} = [\sum_i A_{iy}(1+r_y)^{-i}] / A_y$$

$$(8) \quad D_{Ly} = [\sum_i L_{iy}(1+r_y)^{-i}] / L_y.$$

Finally, let  $D_A$  and  $D_L$  denote the weighted-average dollar/yen duration calculated separately for the assets and liabilities, respectively (recall the assumption  $A=L$ ):

$$(9) \quad D_A = (A_d/A)D_{Ad} + (SA_y/A)D_{Ay}$$

$$(10) \quad D_L = (L_d/A)D_{Ld} + (SL_y/A)D_{Ly}$$

The sufficient and necessary conditions for immunizing an international portfolio are derived as follows.

### First-Order Conditions

Theorem:

(a) The portfolio is immunized against interest rate risk if  $D_A = D_L$ , meaning that the average duration of the assets equals that of the liabilities;

(b) The portfolio is immunized against currency risk if (i)  $A_y = L_y$ , namely the value of the yen assets equals that of the yen liabilities; and (ii)  $D_{Ay} = D_{Ly}$ , namely the duration of the yen assets equals that of the yen liabilities.<sup>2</sup>

Proof: Let the investor's net worth be defined by

$$(11) \quad N = A - L = A_d + SA_y - (L_d + SL_y),$$

and expand (11) as a Taylor series up to the first order for variables  $S$ ,  $r_d$ , and  $r_y$ . Using the definitions in (1)–(4), this expansion yields

$$(12) \quad dN = dS[A_y - L_y] \\ + dr_d[-\sum_i A_{id}(1+r_d)^{-i-1}i + \sum_i L_{id}(1+r_d)^{-i-1}i] \\ + Sdr_y[-\sum_i A_{iy}(1+r_y)^{-i-1}i + \sum_i L_{iy}(1+r_y)^{-i-1}i].$$

The definitions of duration given in (5)–(8) are next substituted in (12)

$$(13) \quad dN = (1+r_d)^{-1}dr_d[-A_dD_{Ad} + L_dD_{Ld}] \\ + S(1+r_y)^{-1}dr_y[-A_yD_{Ay} + L_yD_{Ly}] \\ + dS[A_y - L_y].$$

Given the assumption  $A=L$ , (13) is equivalent to

$$(14) \quad dN = (1+r_d)^{-1}dr_dA[-(A_d/A)D_{Ad} + (L_d/A)D_{Ld}] \\ + S(1+r_y)^{-1}dr_yA[-(A_y/A)D_{Ay} + (L_y/A)D_{Ly}] \\ + dS[A_y - L_y].$$

Given a covered interest rate parity<sup>3</sup> the following relationship between the forward and spot exchange rates must hold

$$(15) \quad S(1+r_d) = F(1+r_y)$$

or equivalently

$$(15') \quad S(1+r_y)^{-1} = F(1+r_d)^{-1}.$$

The first-order Taylor approximation of (15) is

$$(16) \quad dS(1+r_d) + Sdr_d = dF(1+r_y) + Fdr_y.$$

From (15') and (16), we get

$$(17) \quad dr_y S(1+r_y)^{-1} = dr_y F(1+r_d)^{-1} \\ = [Sdr_d + dS(1+r_d) - dF(1+r_y)](1+r_d)^{-1}$$

which is substituted in (14) to obtain

$$(18) \quad dN = (1+r_d)^{-1} dr_d A \{ -(A_d/A)D_{Ad} + (SA_y/A)D_{Ay} \} \\ + [(L_d/A)D_{Ld} + (SL_y/A)D_{Ly}] \\ + [dS(1+r_d) - dF(1+r_y)](1+r_d)^{-1} A \{ -(A_y/A)D_{Ay} + (L_y/A)D_{Ly} \} \\ + dS[A_y - L_y].$$

Equation (18) can be simplified by substituting the definitions of the duration of assets and liabilities given by (9)–(10):

$$(19) \quad dN = (1+r_d)^{-1} dr_d A \{ -D_A + D_L \} \\ + [dS(1+r_d) - dF(1+r_y)](1+r_d)^{-1} A \{ -(A_y/A)D_{Ay} + (L_y/A)D_{Ly} \} \\ + dS[A_y - L_y]$$

thereby completing the proof of this theorem.<sup>4</sup> According to the first line on the right-hand side of (19), if  $D_A = D_L$ , the portfolio is immunized against interest rate risk. According to the second and third lines, if  $A_y = L_y$  and  $D_{Ay} = D_{Ly}$ , then  $dS$  and  $dF$  do not effect the net worth of the portfolio, implying that the portfolio is immunized against currency risk. The hedging of currency risk requires that the duration of assets and liabilities be matched in each currency.

As shown below, when currency risk is considerably greater than interest rate risk, it is possible to assume

$$(20) \quad dS(1+r_d) - dF(1+r_y) \approx 0.$$

With this assumption, the following corollary provides a simpler first-order condition for a successful immunization.<sup>5</sup>

*Corollary: Given the approximation in (20), the portfolio is immunized against both interest rate and currency risks if the following two conditions hold:*

- (a)  $D_A = D_L$
- (b)  $A_y = L_y$ .

*Proof:* Substitute equation (20) in equation (19).

According to this corollary, there may be a complete separation of interest rate risk and currency risk in the immunization process. If (20) holds, the immunization strategy of the multinational firm should be similar to that of a domestic one, requiring only the equality of duration of total assets and liabilities in the two markets combined—without the more stringent condition that the duration of assets and liabilities be matched in each market. The validity of this simplified immunization strategy is further examined against the second-order conditions.

## Second-Order Conditions

We expand  $dN$  as a Taylor series up to the second order:

$$(20) \quad dN = dSN_s + dr_dNr_d + dr_yNr_y \\ + \frac{1}{2}dS^2N_s^2 + \frac{1}{2}dr_d^2Nr_d^2 + \frac{1}{2}dr_y^2Nr_y^2 \\ + \frac{1}{2}dSdr_dNr_{dS} + \frac{1}{2}dSdr_yNr_{yS} + \frac{1}{2}dr_d dr_y Ndr_d r_y$$

where

$$N_s = A_y - L_y = \Sigma A_{iy}(1+r_y)^{-i} - \Sigma L_{iy}(1+r_y)^{-i} \\ Nr_d = -\Sigma A_{id}(1+r_d)^{-i}i + \Sigma L_{id}(1+r_d)^{-i}i \\ Nr_y = S[-\Sigma A_{iy}(1+r_y)^{-i-1}i + \Sigma L_{iy}(1+r_y)^{-i-1}i] \\ N_s^2 = 0 \\ Nr_d^2 = \Sigma A_{id}(1+r_d)^{-i-2}(1+i)i - \Sigma L_{id}(1+r_d)^{-i-2}(1+i)i \\ Nr_y^2 = S[\Sigma A_{iy}(1+r_y)^{-i-2}(1+i)i - \Sigma L_{iy}(1+r_y)^{-i-2}(1+i)i] \\ Nr_{dS} = 0 \\ Nr_{yS} = -\Sigma A_{iy}(1+r_y)^{-i-1}i + \Sigma L_{iy}(1+r_y)^{-i-1}i \\ Nr_d r_y = 0$$

The second-order condition is therefore,

$$(21) \quad \frac{1}{2}dr_d^2 [\Sigma A_{id}(1+r_d)^{-i-2}(1+i)i - \Sigma L_{id}(1+r_d)^{-i-2}(1+i)i] \\ + \frac{1}{2}dr_y^2 S[\Sigma A_{iy}(1+r_y)^{-i-2}(1+i)i - \Sigma L_{iy}(1+r_y)^{-i-2}(1+i)i] \\ + \frac{1}{2}dSdr_y[-\Sigma A_{iy}(1+r_y)^{-i-1}i + \Sigma L_{iy}(1+r_y)^{-i-1}i] > 0.$$

This inequality would be satisfied if, as a sufficient condition, the convexity of both the dollar and yen assets (first and second terms) is greater than that of their corresponding liabilities, and the duration of yen assets is equal to that of yen liabilities (third term). Like the domestic immunization problem, the second-order condition is met if the convexity of total assets is greater than that of total liabilities. But, unlike the domestic immunization problem, the second-order condition also requires that the third term of this inequality—if negative—is smaller in absolute value than the first

two terms, or greater than or equal to zero. The third term is a function of the sign of  $dSdr$  and the duration of liabilities relative to that of assets. Although the presence of this term theoretically restricts the ability to manage separately interest rate risk and currency risk, we confirm below that, practically, the duration of assets and liabilities may not have to be matched in each market. Going back to inequality (21), we further note that if the durations of assets and liabilities are equal: (1) the only requirement set by the second-order condition is that the convexity of the multinational firm's total assets be greater than that of its total liabilities; and (2) according to the first-order condition, both the interest rate risk and currency risk are immunized.

## Practical Considerations

In this section, we conduct two tests designed to examine the empirical viability of the first- and second-order conditions allowing the multinational firm to manage separately the interest rate risk and currency risk. In the first test, we examine the relevant assumption based on the first-order condition in (20) that  $dS(1+r_d)-dF(1+r_y)$  is insignificantly different from zero. In the second test, we examine for various currencies the relevant assumption based on the second-order condition that  $dSdr$  is insignificantly different from zero.

The empirical analysis takes the viewpoint of an investor who has access to Euromarkets, using end-of-month spot rates and Euro interest rates for 1, 3, 6, 12, and 120 months. These rates are used to derive forward rates assuming the covered interest-rate parity expressed by

$$(22) \quad f_{0,n} = S_0(1+r_d)/(1+r_y)$$

where  $n$  denotes the forward contract's time to expiration measured in months, and  $r_d$  and  $r_y$  the respective domestic and foreign interest rates of the same expiration period. The sample includes monthly observations for the period January 1990 through November 1995. The results displayed in Table 1 indicate that in most cases examined here both assumptions are reasonable as both  $dS(1+r_d)-dF(1+r_y)$  and  $dSdr_y$  are insignificantly different from zero at the 5 percent level. The exception is the \$/yen exchange rate for which the assumption does not hold for some maturities. These preliminary results indicate that currency and interest rate risks can sometimes be gainfully managed separately.

## Conclusion

This chapter is concerned with fixed-income asset/liability management of the multinational entity, an environment in which foreign currency risk and interest rate risk are generally presumed to require simultaneous treatment.

**Table 1 Monthly Changes of Spot and Forward Rates (January 1990–November 1995)\***

Currency	Length of Forward Contract	dF(1+r <sub>d</sub> ) -dS(1+r <sub>d</sub> ) (×1,000)	t-value	dSdr (×1,000,000)	t-value
\$/Year	1	0.00057	1.870	-0.00777	-0.889
	3	-0.00067	-3.403	-0.00662	-0.779
	6	-0.00624	-2.849	-0.00647	-0.739
	12	-0.00509	-1.323	-0.00868	-0.850
	120	0.08890	1.331	-0.00868	-0.849
\$/FF	1	0.38131	0.533	0.05684	1.672
	3	1.02409	1.399	0.03106	1.261
	6	1.00676	1.365	0.02026	1.156
	12	0.98220	1.321	0.01088	0.908
	120	-0.87698	0.465	0.01188	0.907
\$/DM	1	1.57421	0.609	-0.00967	-1.226
	3	1.02039	0.386	-0.01013	-1.366
	6	1.09363	0.401	-0.01270	-1.554
	12	1.17745	0.408	-0.01229	-1.123
	120	1.51627	0.144	-0.01228	-1.124
\$/SF	1	3.01459	0.931	-0.01461	-1.536
	3	2.35885	0.706	-0.01460	-1.471
	6	2.45581	0.722	-0.01157	-1.138
	12	2.65194	0.750	-0.00525	-0.433
	120	11.74661	1.303	-0.00520	-0.430

\*  $dF = (f_{0,n} - f_{0,n-1})$  is the monthly change of the forward rate where  $f_{0,n}$  is the forward rate contracted today to expire  $n$  months from now.  $dS = (S_1 - S_0)$  is the monthly change of the spot rate where  $S_0$  and  $S_1$  are the current spot rate and the spot rate a month later, respectively. The  $t$ -value is for rejecting the hypothesis that the average is different from zero at the 5 percent level of significance.

We explore the conditions under which the cost of risk management may be reduced by managing those risks separately. The strategy developed here subject to fairly nonrestrictive assumptions reveals the conditions under which asset and liability duration can be matched separately in each country. Our preliminary empirical investigation shows that those conditions may be frequently met and therefore be worth searching for.

## Endnotes

1. See for example, Fisher and Weil (1971); Yawitz and Marshall (1981); Fong and Vasicek (1983, 1984); Fong and Fabozzi (1985); Maloney and Yawitz (1986); Bierwag (1987); Bierwag, Kaufman, and Latta (1987); Prisman and Shores (1988); Maloney and Logue (1989); Reitano (1992); Fooladi and Roberts (1992); Bierwag, Fooladi, and Roberts (1993).
2. If  $D_{Ay}=D_{Ly}$  and  $D_A=D_L$  then also  $D_{Ad}=D_{Ld}$ . The average duration of the dollar assets is also equal to that of the dollar liabilities.
3. A covered interest rate parity must hold in the absence of arbitrage opportunities when there are no transaction costs. This condition is replaced by two inequalities in the presence of transaction costs.
4. In the absence of foreign currency and foreign interest rate risks, equation (19) is reduced to Redington's (1952) first-order condition.
5. The assumption that  $dS(1 + r_d) - dF(1 + r_f) \approx 0$  is tested in the text that follows. It is shown that this term is insignificantly different from zero for most currencies and maturities studied.

## References

- Bierwag, G. O.; G. G. Kaufman; and A. Toevs. "Immunization Strategies for Funding Multiple Liabilities." *Journal of Financial and Quantitative Analysis* 18, 1983, pp. 113-124.
- Bierwag, G. O. "Bond Returns, Discrete Stochastic Processes and Duration." *Journal of Financial Research* 10, 1987, pp. 191-210.
- Bierwag, G. O.; G. G. Kaufman; and C. Latta. "Bond Portfolio Immunization: Tests of Maturity, One and Two Factor Duration Matching Strategies." *Financial Review* 22, 1987, pp. 203-219.
- Bierwag, G. O.; I. Fooladi; and S. Roberts. "Designing an Immunized Portfolio: Is M-Squared the Key?" *Journal of Banking and Finance* 17, 1993, pp. 1147-1170.
- Fisher, L., and R. L. Weil. "Coping with the Risk of Interest Rate Fluctuations: Returns to Bondholders from Naive and Optimal Strategies." *Journal of Business* 44, 1971.
- Fong, H. G., and F. J. Fabozzi. *Fixed Income Portfolio Management* Homewood, IL: Dow-Jones Irwin, 1985.
- Fong, H. G., and G. A. Vasicek. "Return Maximization for Immunized Portfolio," in *Innovations in Bond Portfolio Management*, G. O. Bierwag, G. Kaufman, and A. Toevs, eds. Greenwich, CN: JAI Press, 1983, pp. 227-238.
- Fong, H. G., and G. A. Vasicek. "A Risk Minimizing Strategy for Portfolio Immunization." *Journal of Finance* 39, 1984, pp. 1541-1546.
- Fooladi, I., and G. S. Roberts. "Bond Portfolio Immunization: Canadian Tests." *Journal of Economics and Business* 44, 1992, pp. 3-17.

Gadkari, V., and M. Spindel. *The Currency Hedging and International Diversification: Implications of a World Reserve Currency Effect*. New York: Salomon Brothers Inc, November 1989.

Hauser, S., and A. Levy. "The Effect of Exchange Rate and Interest Rate Risk on International Currency and Fixed Income Security Allocation." *Journal of Business and Economics* 43, 1991, pp. 375–388.

Hegde, S. P., and K. P. Nunn. "Non-Infinitesimal Rate Changes and Macaulay Duration." *Journal of Portfolio Management* 14, 1988, pp. 69–73.

Leibowitz, M. L.; L. N. Bader; and S. Kogelman. "Global Fixed-Income Investing: The Impact of the Currency Hedge." *Journal of Fixed Income* 3, June 1993, pp. 7–18.

Macaulay, F. R. *Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields, and Stock Prices in the U.S. Since 1856*. New York: National Bureau of Economic Research, 1938.

Maloney, K. J., and J. B. Yawitz. "Interest Rate Risk, Immunization, and Duration." *Journal of Portfolio Management* 12, Spring 1986, pp. 41–48.

Maloney, K. J., and D. E. Logue. "Neglected Complexities in Structured Bond Portfolios." *Journal of Portfolio Management* 15, Winter 1989, pp. 59–68.

Prisman, E. Z., and M. R. Shores. "Duration Measure for Specific Term Structure Estimates and Applications to Bond Portfolio Immunization." *Journal of Finance* 43, 1988, pp. 493–504.

Redington, F. M. "Review of the Principle of Life Office Valuations." *Journal of the Institute of Actuaries* 18, 1952, pp. 286–340.

Reitano, R. R. "Non-Parallel Yield Curve Shifts and Immunization." *Journal of Portfolio Management* 18, Spring 1992, pp. 36–43.

Yawitz, J. B., and W. J. Marshall. "The Shortcomings of Duration as a Risk Measure for Bonds." *Journal of Financial Research* 4, Summer 1981, pp. 91–102.